

Theory and Practice of Representative Sampling

Proper sampling is <u>the</u> critical success factor before analysis and decision making Who / What / When

Kim H. Esbensen

<u>Geological Survey of Denmark & Greenland (GEUS):</u> Research professor (<i>sampling, chemometrics</i>)	2010-2015
Aalborg University (<i>chemometrics, sampling</i>): professor	2001-
Telemark University of Process Technology (HPT): professor	1991-
Norwegian Computing Center & SINTEF: research scientist	1985-
Terra Swede (exploration): geochemical data analyst	1982-
Technical University of Denmark (CTH), Ph.D. (metallurgy, data analysis	s) 1981
University of Aarhus, Denmark: M.Sc. (geology) 1979	



KHE CONSULTING





Theory and Practice of Representative Sampling

Proper sampling the critical success factor before analysis and decision making:

- for the company; for the customer; for the scientist; for the technician
- in science, technology, industry; in the analytical laboratory
- for compliance; for safety; for society
- for the seller; for the buyer; for the middleman, for the arbiter
- for stationary lots and for <u>moving lots</u> (PAT process monitoring & control)









Sampling tool: spear sampler or PAT sensor

Heterogeneity (hidden)





Fundamental Sampling Principle (FSP):

All increments must have the same (non-zero) probability of ending up in the sample









Quick overview of 99 %-ile of PAT "process sampling design"...

Plant sampling



<u>All</u> these designs are "*incorrect*" – sampling process is *non-representative* !

TOS agenda: complete DUALITY



Identical "sampling issues" with or without sensor technologies (PAT) ...



Plant sampling











This sensor is principally "*incorrect*" – sampling process is *non-representative* !

"PROCESS SAMPLING (TOS) – the missing link in PAT"

Kim H. Esbensen & P. Paasch-Mortensen

Bakeev, K. (Ed.) PROCESS ANALYTICAL TECHNOLOGY, <u>2.nd Ed.</u>(2009) (chapter 3)

Process Analytical Technology

Spectroscopic Tools and Implementation Strategies for the Chemical and Pharmaceutical Industries

SECOND EDITION

Editor Katherine A. Bakeev

WILEY

Sampling - looking for unifying principles & actions



PAT priorities (incl. TOS):

- 1. Representativity <u>rules</u>!
- 2. Equipment (TOS correct)
- 3. Data acquisition ...
- 4. Data management ...
- 5. Multivariate calibration
- 6. Proper prediction model validation

Field sampling







Theory and Practice of Representative Sampling

Basic principles and unit operations – an overview Theory of Sampling (TOS)



This is a representative sample ;-)

What are the <u>criteria</u> for representativity?



"SAMPLING – should not be gambling!"

Attributed to: Pierre Gy, founder of Theory of Sampling (TOS)





Theory of Sampling (TOS) – everything in a glance



A most important insight ... where all sampling starts

1. Sampling is <u>never</u> a one-shot operation

2. Sampling is <u>always</u> a multi-stage process:

i) Primary sampling +

ii) Representative Mass-reduction

Princple of Sampling Simplicity (PSI)



Representative Sampling: Theory of Sampling (TOS)

TOS - Axiomatic exposé

Governing Principles (GP) – Sampling Unit Operations (SUO)

- 1. FSP: Fundamental Sampling Principle
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- 9. SUO: Mixing / Blending
- 10. SUO: Representative Mass Reduction (Sub-sampling)



Theory and Practice of Representative Sampling TOS

Material heterogeneity – where it all <u>begins</u> ... and where it may sometimes <u>end</u> as well (fatally)









Large diversity of foods/feed products – HETEROGENEITY rules !!!







How to ensure representative primary sampling for <u>all</u> types of heterogeneous materials?















Remember him? Grab sampling!




There is <u>observable</u> heterogeneity – and there is <u>hidden</u> heterogeneity! This is equally bad w.r.t. the possibility of *proper* representative sampling



But there is a big surprise comming – a feature that will make sampling very, very much easier, <u>despite</u> there being so many very diffrent kinds of materials

Sampling is not really so much material dependent --Sampling is much more up against lot *heterogeneity*

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HETEROGENEITY – the arch enemy



Heterogeneity - the unifying characteristic for all types of material



Grab sampling – its a deadly sin !!



Sampling Unit Operations: Composite Sampling







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1 st Sample Set	2 nd Sample Set
A	В
	С
	D
3 rd Sample Set	4 th Sample set
E	Н
F	I
G	5 th Sample set



Traditional thief sampling (spear sampling) from within pharmaceutical mixing blenders (here a tumbling V-blender) recommends using 10 fixed locations according to a certain order intended to minimize 'drag down' of powder from higher locations.

The fundamental assumption is that these locations (including replication at a few locations) represent the "most in-homogenous" parts of any blend, for all types of mixtures, in all types of blenders.

Alas this strong universal assumption is **not** tenable in the industrial practice.

.. In fact, pretty much <u>everything</u> is wrong with this approach:

• Enormous ISE – ditto sampling bias – totally out of control (spears)

- Even CSE are difficult to control: "one sampling procedure fits all"
- Pharmaceutical mixtures are anything but a similar heterogeneity, or of similar mixing features ...

• Confusion, massive confusion regarding heterogeneity vs. sampling

State-of-the-art approach

ALC:

Camera, WIFI, ...

-

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State-of-the-art approach

Camera, WIFI, ...









Radical solution: IF you have to sample from within a mixer – DO NOT SAMPLE from within a mixer !!





Representative Sampling: Theory of Sampling (TOS)

TOS - Axiomatic exposé

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Exit grab sampling!

The <u>one</u> EXCEPTION from grab sampling

Well there are two exceptions

Oisters - and white wine from the Loire valley

Pierre Gy – founder of the Theory of Sampling (TOS)



Cannes, June 8.th, 2005

Pierre Maurice Gyb. Paris, July 25, 1924

Chem. Eng. Paris Sch. Phys. & Chem. (46) Ph.D. physics. Univ. Nancy (1960) Ph.D. Math. (stat). Univ. Nancy (1975) Gold medal (Soc. l'industri Minerale) (63,76) Lavoisier Medal (Fr. soc. Chemistry) (1995)

... 9 books, 175 papers, 200 lectures ...

Pierre Gy: founder of TOS

A brilliant scientist – a monumental *ouevre* – a gentleman friend to all samplers

From where certain VIKINGS originated who raided *Le Normandie* 1000 years ago and later then setteled there: ... The family Pierre Gy ...

The estuary "Gyland", Flekkefjord, Norway

Lappeenranta University of Technology (LUT) - 1999



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CHEMOMETRICS AND INTELLIGENT LABORATORY **SYSTEMS**

SAMPLING FOR ANALYTICAL PURPOSES

WILEY

PIERRE GY



Theory and Practice of **TOS** Representative Sampling

A minimum understanding of governing principles and sampling unit operations

- for all types of materials (all degrees of heterogeneity:low / intermediate / high
- <u>at all scales</u> (for all lot sizes: small / intermediate / big / extreme)
- unifying principles of <u>representative sampling</u>: field/plant/laboratory



The THEME of today's short course: WHAT comes BEFORE analysis?

.

.

Theory of Sampling (TOS)

Total Sampling Error (TSE)

Measurement Uncertainty (MU)

APGC + MS/MS (single) MS (universal) MS LC-MS + APGC + CI + (tandem) MS GC –GC HPLC / HRMS .. (data independent MS/MS) Tandem Ionization UHPLC / QT [MS/MS libraries] GC – TF-MS

Total Analytical Error (TAE)
Theory of Sampling (TOS)

Measurement Uncertainty (MU)



 $MU_{total} = MU_{sampling} + MU_{analysis}$

Theory of Sampling (TOS) – everything in a glance





Sampling is actually only dependent upon three things

Representative Sampling: Theory of Sampling (TOS)

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Fundamental Sampling Principle (FSP):

All increments must have the same (non-zero) probability of ending up in the sample (non-neg)

FSP





















- the analytical result <u>depends</u> on the sampling procedures used

NB NB NB Can this this be an 'accidental' situation in geo-

science laboratories only?

Whole 12 kg – fully crushed (TOS) compared to 20 g (grab sampling) – missing mass ratio 1:600

IN

But <u>WHAT IF</u> the primary sample material is <u>heterogeneous</u>?

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Sampling in the laboratory: What's the difference w.r.t. the field/plant etc?

Identical sampling issues and problems - at all scales ...

TOS' Simplifying, Governing Principles ...



Principle of Sampling Scale Invariance (SSI)

"1 gram for analysis -----"

How big is the <u>original lot</u>, which shall be characterised by the analytical result?

	Sampling rate:
1 kg	1 / 10 ³
10 kg	1 / 104
100 kg	1 / 10 ⁵
1.000 kg (1 ton)	1 / 10 ⁶
10.000 kg (5 ton)	1 / 107

......

GRAPHIC illustration (trace concentrations)

- trace concentration: below 1%









beware of spoon size



beware of spoon size



beware of spoon size





Experiment exemplum - credits: Suzanne Roy & Lars Beck



OHeterogeneity as a function of:

OConcentration OSpatial distribution

OSampling tool size



These sampling tool sizes are very unrealistic w.r.t. real-life situations ...

Sampling rate - 1:50



Petri Spinner

18 cm² per revolution 18 cm² per analysis 35 gram per sample



Bottle Spinner

15 cm² per revolution 28 cm² per analysis 50 gram per sample



Spiral Spinner 374 cm² per revolution 110 cm² per analysis 600 gram per sample



Theory of Sampling (TOS) – everything in a glance



- the analytical result <u>depends</u> on the sampling procedures used





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Composite sampling employs Q increment extractions with the aim to 'cover' the lot volume (only Q = 4 increments shown in this principal illustration). Proportional to the heterogeneity encountered, a higher number of increments will be required. Comp samp. must always respect FSP !!!

GP (6) Lot Heterogeneity Characterisation (LHC) guarantees that no sampling plan, sampling procedure nor sampling equipment is employed without a mandatory heterogeneity characterisation of the lot material.

Composite sampling is specifically demanding that grab sampling (extraction of one single increment only) is never invoked, unless thoroughly tested and accepted by either a Replication Experiment (RE) or by variographics.









Crushing (comminution) is a sampling unit operation which is only brought to bear when necessary, i.e. when the top particle size is contrasting too much with respect to smaller size ranges in order for sampling to be effective and representative. Comminution is the technical process in which the top particle sizes is preferentially crushed first.

A consequence of crushing/comminution is that the majority of particle sizes tend to become more similar, with the further advantage that mixing becomes more effective.



ceration, crushing or shredding in the presense of a ilitating liquid (often used for selective extraction), as olied to biological materials also lead to reduced general rticle sizes.



Mixing is a forced mechanical process designed to reduce the distributional heterogeneity (DH) of a material system.

It is always advantageous to mix the results of a sampling or a sub-sampling process *before* further processing (subsampling or a next stage mass reduction).

X

Blending is mixing under stoichiometric constraints, i.e. the final mixing product, a blend, must satisfy compositional constraints e.g. tea, tobacco, cement, pharmaceutical drugs.

Mixing / blending can be applied to both polyphase dry systems (aggregates) and to slurries (solid – liquid systems).









Representative Mass Reduction (RMR) is the key sampling unit operation connecting all sampling stages. Often the terms mass reduction and sub-sampling are used inter alia. There are very many sub-sampling procedures and types of equipment offered on the market, but far from all deliver representative solutions.

For stationary lots, the benchmark study by Petersen et al. (2004) showed conslusively that only the *riffle-splitting principle* lead to Representative Mass Reduction (RMR). Riffle splitters have different physical manifestations; both stationary and roraty solutions exist.

For dynamic lots, lots in movement, the *Vezin sampler* is by far the most effective, fully representative RMR equipment in existence. The Vezin sampler is also superior regarding slurries a.o.

Representative Sampling: Theory of Sampling (TOS)

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Sampling Unit Operations: Composite Sampling





Theory of Sampling (TOS) – everything in a glance



Representative Sampling: Theory of Sampling (TOS)

TOS - Axiomatic exposé

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Crushing / Maceration

REALLY difficult materials ... Que faire?



Theory of Sampling (TOS) – everything in a glance



Sampling Unit Operations: Mixing / Blending

A pro pos CRUSHING + MIXING before further sub-sampling:

















Particle size reduction (crushing)

var(FSE) =

error vs sample mass



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Sampling Unit Operations: Mixing / Blending



Mixing – intuitively clear. However, forceful mixing is a much less effective process than commonly known ...

Mixing – It is manifestly not enough just "to mix" and then hope all is OK



Mixing – It is always necessary to conduct a baseline *validation* of the mixing/blending operation in use, e.g. A Replication Experiment (RE).

Theory of Sampling (TOS) – everything in a glance



Sampling Unit Operation: Mixing / Blending

Reduces contributions to sampling variation from the Grouping and Segregation Error (GSE)

$$s^{2}(GSE) = \zeta \cdot \gamma \cdot s^{2}(FSE) \qquad \gamma = \frac{N_{F} - N_{G}}{N_{G} - I} \approx \frac{N_{F}}{N_{G}}$$

Segregation factor \checkmark Grouping factor (unaffected by mixing - reduced only by selecting smaller increments) $\zeta \approx 0$

Four practical Sampling Unit Operations (SUO)

- 1. Composite Sampling
- 2. Particle Size Reduction (comminution)
- 3. Mixing / blending
- 4. Representative Mass Reduction (- sample preparation)



Used as active steps in the sampling process (often used several times, in combination)

Fundamental insights - I

O The empowering role of universal principles & SUO's

Governing principles (GP) & Sampling Unit Operations (SUO)

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- 10. SUO: Representative Mass Reduction (sub-sampling/splitting)

All GP's & SUO's are <u>not</u> involved in <u>all</u> sampling tasks. The analysis & the sampling objective determines which GP's and SUO's to use. The Theory of Sampling (TOS) to the fore ... DS 3077: First horisontal standard (2013)

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And Road		Internet
-000-000P		

Dansk standard

DS 3077 2. udgave 2013-08-26

Repræsentativ prøvetagning – Horisontal standard

Representative sampling - Horizontal standard







This standard outlines a practical, iterative, self-controlling approach with minimal complexity, based on the Theory of Sampling (TOS) The generic sampling process described and all elements involved are sufficient and necessary for the stated objective, with the consequence that no exceptions can be allowed in order to be able to document the intended sampling representativity. It is necessary to consider the full pathway from primary sampling to analytical results in order to be able to guarantee a reliable and valid analytical outcome. This standard, including normative references, annexes (and further, optional references) constitute a complete and sufficient competence basis for this purpose. The present approach will ensure appropriate levels of accuracy and precision for both primary sampling as well as for all sub-sampling procedures and mass-reduction systems at the subsequent laboratory stages before analysis.

An important analogy (for some ...)

1.
$$\nabla \cdot \mathbf{D} = \rho_V$$

2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

Maxwell's Equations describe the world of electromagnetics. The four equations describe how electric and magnetic fields *propagate*, *interact*, and how they are *influenced* by objects





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"Representative Sampling for Food and Feed Materials"

World's first attempt to bring the Theory of Sampling (TOS) and foods/feed safety assessment together!



Transatlantic Special Section taskforce, Oct. 2014, Windsor, CO: *Nancy Thiex, Kim H. Esbensen, Charles A. Ramsey, Claas Wagner, Claudia Paoletti.*

Fundamental insights - II

• The role of *statistics* – in sampling and analysis

A (most) surprising and critical distinction

Because the sampler is free to choose between alternative sampling procedures, - equipment and – conditions

 based, or <u>not</u> based, on a competent understading and accept of the adverse role of heterogeneity when <u>interacting</u> with a specific sampling process

the analytical result will <u>depend</u> on these choices!

All analytical result will depend on the specific sampling procedure(s) used to deliver the analytical aliquot!



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Lot Dimensionality (definition via practical increments)

No correlation exists between increments. Total access to the complete lot volume. Increments can be picked at will without untoward effort or difficulty (practical def)

Increments only cover *two* lot dimensions (e.g. a cross-wise planar-parallel "slice")

Increments only cover *one* lot dimension (e.g. a drill-core; a profile)

Increments do <u>not</u> fully cover <u>any</u> of the lot dimensions (e.g. a "depth sample")



LDT: Lot Dimensionality Transformation





Fundamental Sampling Principle (FSP): 1-D lots



Fundamental Sampling Principle (FSP):

All increments must have the same (non-zero) probability of ending up in the sample

EXAMPLE: lot dim. transformation (LDT) Ceiinal sanoing local stion: I.D.Hansportation toother recent Q sampling site: 1-D

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The analytical process always contains several sampling and preparation steps, but <u>usually</u> primary sampling <u>dominates</u>

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Theory of Sampling (TOS)

Measurement Uncertainty (MU)



 $MU_{total} = MU_{sampling} + MU_{analysis}$
Fundamental insights - III

- The empowering role of universal principles & SUO's –
- O Sampling of <u>stationary lots</u> vs. <u>moving lots</u> (process sampling)

 Illustration of the origin of Incorrect Sampling Errors (ICS) and their effects (unnecessary, but fatal bias)









Fundamental insights - IV

O The most important distinction: analysis vs. sampling bias

Critical distinction between analysis (*sensu strictu*) – and sampling_plus_analysis! MUST be understood!



Sampling process: BIAS + imprecision - - TOS: a varying, an inconstant bias!



Analytical process: bias + imprecision (statistical concept: a constant bias)

Representative Sampling: Theory of Sampling (TOS)

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- 6. The Replication Experiment, RE(**r**)
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- 9. SUO: Mixing / Blending
- 10. SUO: Representative Mass Reduction (Sub-sampling)

The Replication Experiment, RE(r)

HOW TO? - What you can do one time:

maat

-you can replicate, f.eks. 10 times

Replication Experiments need <u>only</u> to be carried out for *each principal* type of material as sampled by a *specific* sampling procedure ... NB NB NB NB 35744

Relative Sampling Variance $(RSV) - C.V._{(rel)}$

SS

TS

PS

Relative Sampling Variability (RSV):

 $C.V._{(rel)} = [STD / X_{avr.}] \times 100$

Relative Sampling Variability (RSV), aka

relative Coefficient of Variation, C.V.(rel)

Replication Experiment: RSV[%] characterises <u>all</u> sampling + analysis operations – for all types of equipment and all types of materials !!!

















CV₁ - Variability among analytical replicates = analytical uncertainty

CV₂ - Variability among increments = analytical + sampling uncertainty

Lot	CV₁ ←	\rightarrow CV ₂
1	10,18	116,85
2	7,77	44,5
3	6,85	40,8
4	6,52	51,59
5	22,34	249,88
6	30,96	133,64
7	20,12	60,63
8	19,65	57,54
9	5,06	39,74
10	6,52	62,13
11	5,68	52,88
12	6,25	38,04
13	4,69	35,03
14	15,48	218,12
15	34,09	171,81

There never was an analytical result – <u>without</u> preceding sampling

The specific analytical mesurement uncertainty CV_1 should not be used as a measure of the <u>total</u> measurement uncertainty CV_2 .

What would be the consequence(s)?

Who is responsible?









Representative Sampling: Theory of Sampling (TOS)

TOS - Axiomatic exposé

Governing principles (GP) – Sampling unit operations (SUO)

- 1. FSP: Fundamental Sampling Principle
- 2. SSI: Sampling Scale Invariance
- 3. PSC: Sampling Correctness (bias-free sampling)
- 4. PSS: Sampling Simplicity (primary sampling + mass-reduction)
- 5. LDT: Lot Dimensionality Transformation
- 6. LHC: Lot Heterogeneity Characterization (0-D, 1-D)
- 7. SUO: Composite Sampling
- 8. SUO: Comminution
- 9. SUO: Mixing / Blending
- 10. SUO: Representative Mass Reduction (Sub-sampling)

TOS in the laboratory – TOS in the laboratory

References

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http://www.impublications.com/tos-forum

http://www.spectroscopyeurope.com/articles/sampling

Never hesitate to contact experts, colleagues, friends re. TOS





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Thank you for your attention!

Fundamental insights -

○ …a HUGE business opportunity …

Can be discussed later



Advanced stuff – only if there is time

Effect of problem	Causes: sampling variation FSE GSE IEE IPE IWE TAE	IDE (cyclicity)	
Sampling Errors (ISE) TOS' Preventive Paradigm (TPP)	(trends)	Bias Test(s) Round Robin test(s) "Correction factors" "Effects estimation"	
Principle of Sampling Correctness (PSC)	Cour	t of fixing problem	ants



<u>Top priority</u> problems	Problems to be Solved <u>after</u> Performing a Feasibility study
Low priority problems	Problems that are <u>not worth</u> too much attention

Cost of fixing problem

Courtesy: Francis Pitard Sampling Consultants

A hidden elephant in the room ...

• - a fundamental <u>difference</u> of critical importance:

○ a <u>sample cell</u> is containing the sample to do analysis







A hidden elephant in the room ...

• - a fundamental <u>difference</u> of critical importance:

• a <u>sampling cell</u> is sampling <u>and</u> doing the analysis





"Sampling valves" – only one is TOS-representative





The multivariate data matrix: embedded errors in all cells

199.1	205.6	210.7	213.2	216.0	200.6	208.5	212.6	216.5	218.5	202.4	209.9	214.8	217.9	220.4	4.1	4.5	4.8
198.5	204.1	210.0	212.0	214.6	199.4	206.3	211.9	215.1	217.5	202.0	208.8	214.5	217.7	218.9	4.1	4.4	4.7
194.6	200.7	205.4	207.2	209.0	198.1	204.9	209.3	211.5	214.3	201.3	207.6	213.4	215.9	218.0	3.9	4.2	4.5
194.8	200.1	206.8	210.3	212.6	196.4	204.1	210.0	213.4	216.6	198.6	206.6	213.0	216.1	219.2	1.7	1.9	2.0
194.1	201.3	207.1	211.3	213.7	196.6	261.9	210.9	215.1	217.5	199.8	208.1	213.7	218.7	220.2	1.9	2.2	2.4
193.2	201.2	206.9	210.1	213.2	197.0	204.9	210.1	214.2	217.8	199.5	207.1	214.3	218.6	219.9	1.7	2.0	2.3
193.8	200.4	205.6	208.4	209.5	196.8	203.4	208.8	213.5	214.3	198.5	206.2	212.8	215.7	216.9	2.2	2.4	2.5
193.5	201.5	205.3	208.0	211.1	195			12.0	214.4	199.2	205.6	212.4	215.1	217.9			
194.6	202.0	207.1	209.8	212.2	197			13.5	21.5 2	200.1	206.5	212.9	216.7	218.8			
179.5	187.2	192.8	194.7	197.4	185			01.5	203.9	129	196.3	202.2	205.3	207.3			
178.4	185.9	189.8	193.8	195.7	186			02.4	203.3	189.0	197.9	202.1	206.0	208.6			
101 /	188.8	102.7	105.0	107.0	107			0						4			
101.4	100.0	132.7	190.9	197.0	107		K		T .		1	• 4 1		4			
194.0	199.7	205.5	208.3	210.7	196			1 E	low	to w	ork v	with	such				
194.0 195.2	199.7 201.3	205.5 207.4	208.3 209.6	210.7 212.7	196 198				Iow rror	to w s? (T	ork v SE + '	with	such	L (E) ⁴			
194.0 195.2 193.8	199.7 201.3 200.8	205.5 207.4 206.2	208.3 209.6 210.1	210.7 212.7 211.7	196 198 198	Ζ		¹ E ¹ e	Iow rror	to w s? (T	ork v SE +	with TAE -	such + CM	L 0 E) 0			
194.0 195.2 193.8 189.9	199.7 201.3 200.8 197.9	205.5 207.4 206.2 202.7	208.3 209.6 210.1 207.4	210.7 212.7 211.7 209.6	196 198 196 193	Ζ		1 E 1 e 11.5	Iow rror 213.4	to w s? (T 195.9	ork v SE + 204.8	with TAE - 211.3	such + CM 214.6	E) 0 218.0			
194.0 195.2 193.8 189.9 193.4	199.7 201.3 200.8 197.9 201.6	205.5 207.4 206.2 202.7 206.9	208.3 209.6 210.1 207.4 210.4	210.7 212.7 211.7 209.6 213.7	196 198 196 193 195	Δ.		1 E 1 e 11.5 14.7	Iow rror 213.4 217.3	to w s? (T 195.9 199.3	ork SE + 204.8 206.9	with TAE - 211.3 214.2	such + CM 214.6 217.1	E) 0 218.0 219.8			
191.4 194.0 195.2 193.8 189.9 193.4 193.9	199.7 201.3 200.8 197.9 201.6 200.9	205.5 207.4 206.2 202.7 206.9 207.7	208.3 209.6 210.1 207.4 210.4 210.9	210.7 212.7 211.7 209.6 213.7 213.4	196 198 196 193 195 195	205.6	211.5	1 E 1 E 11.5 14.7 215.4	Iow rror 213.4 217.3 218.4	to w s? (T 195.9 199.3 200.9	O rk SE + 204.8 206.9 209.0	with TAE - 211.3 214.2 216.0	such + CM 214.6 217.1 219.0	E) 218.0 219.8 223.0	1.6	1.9	2.1
191.4 194.0 195.2 193.8 189.9 193.4 193.9 200.5	199.7 201.3 200.8 197.9 201.6 200.9 206.8	205.5 207.4 206.2 202.7 206.9 207.7 212.1	208.3 209.6 210.1 207.4 210.4 210.9 214.2	210.7 212.7 211.7 209.6 213.7 213.4 217.3	196 198 196 193 195 195 197.5 202.7	205.6 209.8	211.5 214.7	1 E 1 e 11.5 14.7 215.4 217.8	Low rror 213.4 217.3 218.4 220.8	to w s? (T 195.9 199.3 200.9 204.0	ork SE + 204.8 206.9 209.0 212.2	with TAE - 211.3 214.2 216.0 216.5	such + CM 214.6 217.1 219.0 220.3	E) 0 218.0 219.8 223.0 224.0	1.6 3.2	1.9 3.3	2.1 3.5
191.4 194.0 195.2 193.8 189.9 193.4 193.9 200.5 193.3	199.7 201.3 200.8 197.9 201.6 200.9 206.8 201.2	205.5 207.4 206.2 202.7 206.9 207.7 212.1 206.5	208.3 209.6 210.1 207.4 210.4 210.9 214.2 209.3	210.7 212.7 211.7 209.6 213.7 213.4 217.3 211.9	196 198 198 193 195 197.5 202.7 196.7	205.6 209.8 203.6	211.5 214.7 210.4	1 E 1 E 11.5 14.7 215.4 217.8 213.8	Low 213.4 217.3 218.4 220.8 216.5	to w s? (T 195.9 199.3 200.9 204.0 198.5	SE + 204.8 206.9 209.0 212.2 206.1	with TAE - 211.3 214.2 <u>216.0</u> 216.5 214.0	such + CM 214.6 217.1 219.0 220.3 216.3	E) 4 218.0 219.8 223.0 224.0 218.8	1.6 3.2 1.9	1.9 3.3 2.2	2.1 3.5 2.4
191.4 194.0 195.2 193.8 189.9 193.4 193.9 200.5 193.3 189.9	199.7 201.3 200.8 197.9 201.6 200.9 206.8 201.2 196.3	205.5 207.4 206.2 202.7 206.9 207.7 212.1 206.5 202.3	208.3 209.6 210.1 207.4 210.4 210.9 214.2 209.3 205.4	210.7 212.7 211.7 209.6 213.7 213.4 217.3 211.9 208.3	196 198 198 193 195 197.5 202.7 196.7 192.9	205.6 209.8 203.6 201.3	211.5 214.7 210.4 207.1	1 E 1 E 11.5 14.7 215.4 217.8 213.8 210.7	Low 213.4 217.3 218.4 220.8 216.5 214.1	to we s? (T 195.9 199.3 200.9 204.0 198.5 194.9	SE + 204.8 206.9 209.0 212.2 206.1 204.0	211.3 214.2 216.0 216.5 214.0 210.4	such 214.6 217.1 219.0 220.3 216.3 213.6	E) 0 218.0 219.8 223.0 224.0 218.8 216.3	1.6 3.2 1.9 1.3	1.9 3.3 2.2 1.5	2.1 3.5 2.4 1.7
101.4 194.0 195.2 193.8 189.9 193.4 193.9 200.5 193.3 189.9 180.7	199.7 201.3 200.8 197.9 201.6 200.9 206.8 201.2 196.3 186.5	205.5 207.4 206.2 202.7 206.9 207.7 212.1 206.5 202.3 192.1	208.3 209.6 210.1 207.4 210.4 210.9 214.2 209.3 205.4 194.4	210.7 212.7 211.7 209.6 213.7 213.4 217.3 211.9 208.3 197.5	196 198 198 193 195 197.5 202.7 196.7 192.9 186.6	205.6 209.8 203.6 201.3 194.6	211.5 214.7 210.4 207.1 199.5	1 1 1 1 11.5 14.7 215.4 217.8 213.8 210.7 202.8	Low 213.4 217.3 218.4 220.8 216.5 214.1 204.8	to we s? (T 195.9 199.3 200.9 204.0 198.5 194.9 191.1	SE + 204.8 206.9 209.0 212.2 206.1 204.0 198.8	211.3 214.2 216.0 216.5 214.0 210.4 205.1	such 214.6 217.1 219.0 220.3 216.3 213.6 207.5	E) 0 218.0 219.8 223.0 224.0 218.8 216.3 210.6	1.6 3.2 1.9 1.3 0.9	1.9 3.3 2.2 1.5 1.2	2.1 3.5 2.4 1.7 1.4
101.4 194.0 195.2 193.8 189.9 193.4 193.9 200.5 193.3 189.9 180.7 193.3	199.7 201.3 200.8 197.9 201.6 200.9 206.8 201.2 196.3 186.5 200.0	205.5 207.4 206.2 202.7 206.9 207.7 212.1 206.5 202.3 192.1 206.5	208.3 209.6 210.1 207.4 210.4 210.4 210.9 214.2 209.3 205.4 194.4 210.1	210.7 212.7 211.7 209.6 213.7 213.4 217.3 211.9 208.3 197.5 212.0	196 198 198 193 195. 197.5 202.7 196.7 192.9 186.6 195.9	205.6 209.8 203.6 201.3 194.6 203.2	211.5 214.7 210.4 207.1 199.5 209.9	1 E 1 E 1 E 11.5 14.7 215.4 217.8 213.8 210.7 202.8 212.3	Low 213.4 217.3 218.4 220.8 216.5 214.1 204.8 215.7	to we s? (T 195.9 199.3 200.9 204.0 198.5 194.9 191.1 197.2	SE + 204.8 206.9 209.0 212.2 206.1 204.0 198.8 205.2	vith TAE - 211.3 214.2 216.0 216.5 214.0 210.4 205.1 212.0	such 214.6 217.1 219.0 220.3 216.3 213.6 207.5 215.8	E) 0 218.0 219.8 223.0 224.0 218.8 216.3 210.6 218.2	1.6 3.2 1.9 1.3 0.9 1.9	1.9 3.3 2.2 1.5 1.2 2.1	2.1 3.5 2.4 1.7 1.4 2.2
101.4 194.0 195.2 193.8 189.9 193.4 193.9 200.5 193.3 189.9 180.7 193.3 194.4	199.7 201.3 200.8 197.9 201.6 200.9 206.8 201.2 196.3 186.5 200.0 201.5	205.5 207.4 206.2 202.7 206.9 207.7 212.1 206.5 202.3 192.1 206.5 208.3	208.3 209.6 210.1 207.4 210.4 210.9 214.2 209.3 205.4 194.4 210.1 211.8	210.7 212.7 211.7 209.6 213.7 213.4 217.3 211.9 208.3 197.5 212.0 215.1	197 196 198 195 195 197.5 202.7 196.7 192.9 186.6 195.9 197.4	205.6 209.8 203.6 201.3 194.6 203.2 205.5	211.5 214.7 210.4 207.1 199.5 209.9 212.4	1 E 1 E 1 E 11.5 14.7 215.4 217.8 213.8 210.7 202.8 212.3 216.0	Low 213.4 217.3 218.4 220.8 216.5 214.1 204.8 215.7 218.1	to we s? (T 195.9 199.3 200.9 204.0 198.5 194.9 191.1 197.2 200.4	SE + 204.8 206.9 209.0 212.2 206.1 204.0 198.8 205.2 208.3	211.3 214.2 216.0 216.5 214.0 210.4 205.1 212.0 215.9	such 214.6 217.1 219.0 220.3 216.3 213.6 207.5 215.8 218.5	E) 0 218.0 219.8 223.0 224.0 218.8 216.3 210.6 218.2 218.2 218.2	1.6 3.2 1.9 1.3 0.9 1.9 1.9 1.4	1.9 1.9 3.3 2.2 1.5 1.2 2.1 1.9	2.1 3.5 2.4 1.7 1.4 2.2 2.2



Multivariate Data Models *aka* "Bilinear Data Models" (PCA, PLS. SIMCA)








Clearly it gets more and more <u>realistic</u> to validate such disparate data structures !

But this is exactly what X-validation <u>connot</u> do – there is only ONE data set, training set!



Model complexity: #



Model complexity: a

"Prediction precision is not all that good. But more samples will make the prediction more precise"

Predicted

Reference

Θ

e

It will not! More samples will simply reflect the same underlying heterogeneity

Predicted

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÷

Reference

Special Issue Article

CHEMOMETRICS

Received: 24 January 2010,

Revised: 21 February 2010,

Accepted: 23 February 2010,

(www.interscience.wiley.com) DOI: 10.1002/cem.1310

See literature ;-) Principles of Proper Validation: use and abuse of re-sampling for validation

Kim H. Esbensen^{a*} and Paul Geladi^b

The final word !!!

Validation in chemometrics is presented using the exemplar context of multivariate calibration/prediction. A phenomenological analysis of common validation practices in data analysis and chemometrics leads to formulation of a set of generic Principles of Proper Validation (PPV), which is based on a set of characterizing distinctions: (i) Validation cannot be understood by focusing on the methods of validation only; validation must be based on full knowledge of the underlying definitions, objectives, methods, effects and consequences—which are all outlined and discussed here. (ii) Analysis of proper validation objectives implies that there is one valid paradigm only: test set validation. (iii) Contrary to much contemporary chemometric practices (and validation myths), cross-validation is shown to be unjustified in the form of monolithic application of a one-for-all procedure (segmented cross-validation) on all data sets. Within its own design and scope, cross-validation is in reality a sub-optimal simulation of test set validation, crippled by a critical sampling variance omission, as it manifestly is based on one data set only (training data set). Other re-sampling validation methods are shown to suffer from the same deficiencies. The PPV are universal and can be applied to all situations in which the assessment of performance is desired: prediction-, classification-, time series forecasting-, modeling validation. The key element of PPV is the Theory of Sampling (TOS), which allow insight into all variance generating factors, especially the so-called incorrect sampling errors, which, if not properly eliminated, are responsible for a fatal inconstant sampling bias, for which no statistical correction is possible. In the light of TOS it is shown how a second data set (test set, validation set) is critically necessary for the indusion of the sampling errors incurred in all 'future' situations in which the validated model must perform. Logically, therefore, all one data set re-sampling approaches for validation, especially cross-validation and leverage-corrected validation, should be terminated, or at the very least used only with full scientific understanding and disclosure of their detrimental variance omissions and consequences. Regarding PLS-regression, an emphatic call is made for stringent commitment to test set validation based on graphical inspection of pertinent t-u plots for optimal understanding of the X-Y interrelationships and for validation guidance. QSAR/QSAP forms a partial exemption from the present test set imperative with no generalization potential. Copyright © 2010 John Wiley & Sons, Ltd.

Keywords: Principles of Proper Validation (PPV); future performance assessment; test set validation; cross-validation; re-sampling; predictive regression; Theory of Sampling (TOS)

Published online in W

Never be afraid to contact experts, colleagues, friends re. TOS



Sampling Principles: LHC (3-D vs. 1-D cases)

Lot Heterogeneity Characterization (LHC)

- » Why? It makes *no sense* to design a sampling procedure without knowing the lot heterogeneity quantitatively
- » A *replication experiment* will reveal TSE as well as the sampling steps generating the largest variation
- » 10 (40,60) samples provides all the necessary information:
- » Complete empirical sampling error estimation –
 VARIOGRAPHIC ANALYSIS (

(1-D case)

Sampling Principles: LHC (3-D vs. 1-D cases)

SUO: Variography (variographics)

- 1-dimensional heterogeneity characterization
- 1-D lots: processes, long stationary piles, ordered series etc.



Valuable information about lot *variation* (trends, upsets, periodic phenomena ...)

Sampling Principles: LHC (3-D vs. 1-D cases)

SUO: Variography

"Lag" is the distance between increments (samples) along the time or spatial dimension

A "variogram" displays the total "variation" as a function of the "lag"



	В	С	D	E	F	G	н	1	J	K	L	M
1	Hq (Pred Values)	Lag (j)	V(j)	Q-j	1/(2(Q-j))	sum(hq+j-hq)^2	Lag1	Lag2	Lag3	Lag4	Lag5	Lag6
2												
3	15.04	1	0.245	207	0.002	102	0.00214	0.21641	0.15936	0.04268	0.64609	2.13949
4	14.99	2	0.409	206	0.002	169	0.17548	0.12454	0.06396	0.57381	2.00619	1.34351
5	14.57	3	0.455	205	0.002	186	0.00436	0.45132	0.11465	0.99501	0.5479	1.09161
6	14.64	4	0.490	204	0.002	200	0.36699	0.1637	1.13103	0.64996	0.95805	3.47114
7	15.24	5	0.573	203	0.002	233	1.02091	2.78656	1.99374	0.13913	1.5808	0.07054
8	14.23	6	0.638	202	0.002	258	0.43415	0.16128	1.9138	5.14246	1.62818	0.74909
9	13.58	7	0.652	201	0.002	262	0.0662	4.17099	8.56499	3.74384	2.3238	3.2436
10	13.83	8	0.668	200	0.003	267	3.18623	7.12516	2.81434	1.60554	2.38301	5.35876
11	15.62	9	0.699	199	0.003	278	0.78199	0.01153	0.26822	0.05823	0.28079	1.30599
12	16.50	10	0.730	198	0.003	289	0.98347	1.96616	1.26698	0.1256	4.10913	6.36351
13	15.51	11	0.699	197	0.003	275	0.16851	0.01793	0.40615	1.07205	2.34365	2.22189
14	15.10	12	0.708	196	0.003	277	0.07651	1.09788	0.3905	1.2553	1.16662	1.67314
15	15.38	13	0.745	195	0.003	291	0.59475	0.8127	1.95161	1.84063	2.46521	3.23137
16	16.15	14	0.750	194	0.003	291	2.79793	4.70109	4.52796	5.48169	6.59873	3.62027

 $V(j) = \frac{1}{2(Q_{total} - j)} \sum_{q=1}^{Q_{total} - j} \left(\frac{h_{q+j} - h_q}{p_{q+j} - p_q} \right)^2$







Extracting TSE+TAE information from real time measurements through variographic analysis









From raw data to variogram

Outliers significantly afect the variogram.

ALL outliers should be removed - sequentially



Sampling Principles: LHT (1-D case)

Variographics:

10.200

10.000

9.800 9.600

9.400

9.200

9.000

8.800

8.600

8.400

8.000 7.800

7.600

7.400

7.000

6.800 6.600

Analytical Concentration

Ribe Biogas powerplant (daily sampling)

Analytical Concentration vs. Increment Number



cycle with

period 7 lags

Analytical grade (CH₄ yield m³)

5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115

Increment Number

Variogram and characteristic features

Decreasing Total Sampling Errors (1-D lots) Sampling error plots

Total Sampling Error - f(V(j), Q, j)



Lag↓, sampling rate ↑ TSE↓

Direction of steepest error descent ... Conditioned on a <u>preselected</u> error level